

A Process Calculus for Expressing Finite Place/Transition Petri Nets

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We introduce the process calculus Multi-CCS, which extends conservatively CCS with an operator of strong prefixing able to model atomic sequences of actions as well as multiparty synchronization. Multi-CCS is equipped with a labeled transition system semantics, which makes use of a minimal structural congruence. Multi-CCS is also equipped with an unsafe P/T Petri net semantics by means of a novel technique. This is the first rich process calculus, including CCS as a subcalculus, which receives a semantics in terms of unsafe, labeled P/T nets. The main result of the paper is that a class of Multi-CCS processes, called *finite-net processes*, is able to represent all finite (reduced) P/T nets.

1 Introduction

Labeled transition systems with finitely many states and transitions can be expressed by the CCS [18] sub-calculus of *finite-state processes*, i.e., the sequential processes generated from the empty process 0 , prefixing $\mu.p$, alternative composition $p_1 + p_2$ and a finite number of process constants C , each one equipped with a defining equation $C \stackrel{def}{=} p$. Intuitively, each state s_i is modeled by a constant C_i , whose defining equation contains one summand $a_j.C_j$ for each transition leaving state s_i labeled by action a_j and reaching the state s_j . This celebrated result of Milner offers a process calculus to express, up to isomorphism, all finite-state labeled transition systems. The main advantage of this result is that (i) finite-state lts's can be defined compositionally, and (ii) behavioral equivalences over finite-state lts's can be axiomatized [19].

This paper addresses the same language expressibility problem for finite labeled Place/Transition Petri nets without capacity bounds on places. We single out a fragment of an extension of CCS, called Multi-CCS, such that not only all processes of this fragment generate finite P/T nets, but also for any finite (reduced) P/T net we can find a term of the calculus that generates it. This solves the open problem of providing a process calculus for general Petri nets. and opens interesting possibilities of cross-fertilization between the areas of Petri nets and process calculi. In particular, it is now possible, on the one hand, (i) to define any finite P/T net compositionally and (ii) to start the investigation of axiomatization for behavioral equivalences over such a large class of nets; on the other hand, it is now possible (iii) to reuse all the techniques and decidability results available for P/T nets also for (this fragment of) Multi-CCS, as well as (iv) define non-interleaving semantics, typical of Petri nets, also for Multi-CCS.

We equip Multi-CCS with an operational net semantics that takes inspiration from Goltz's idea of using unsafe, labeled P/T nets [8, 9, 10] for a CCS subcalculus without restriction, and Busi & Gorrieri net semantics for π -calculus [3], where however inhibitor arcs are used to model restriction. The extension of the approach to restriction and strong prefixing is not trivial and passes through the introduction of an auxiliary set of *restricted* actions and the definition of a suitable notion of syntactic substitution. We prove a soundness result, i.e., p and $Net(p)$ are strongly bisimilar, where the net $Net(p)$ is the subnet reachable from the marking associated to process p .

The Multi-CCS sub-calculus of *finite-net processes* is generated as follows:

$$\begin{aligned} s &::= \mathbf{0} \mid \underline{\mu}.t \mid \underline{\underline{\mu}}.t \mid s + s \\ t &::= s \mid t \mid t \mid C \\ p &::= t \mid (va)p \mid p \mid p \end{aligned}$$

where the operator $\underline{\mu}.t$, called *strong prefixing* (in opposition to normal prefixing), expresses that action μ is the initial part of an atomic sequence of actions that continues with t . This operator, introduced in [12], is also at the base of multiparty synchronization, obtained as an atomic sequence of binary CCS-like synchronizations. As a constant $C \stackrel{def}{=} t$, we have that parallel composition \mid may occur inside the body t of recursively defined constants; hence, finite-net processes are infinite-state processes. On the contrary, restriction (va) is not allowed in the body of recursively defined constants. We also require that the alternative composition $+$ is guarded, i.e., all summands are sequential. Finally, constants are assumed to be *guarded*, i.e., in any defining equation each occurs inside a *normally prefixed* subprocess $\underline{\mu}.t$.

We prove that the operational net semantics associates a finite P/T net $Net(p)$ to any finite-net process p . Conversely, we also prove that for any finite reduced P/T net N , we can find a finite-net process p_N such that $Net(p_N)$ and N are isomorphic. The construction of the finite-net process p_N from a net N associates to each place s_i of the net a process constant C_i , whose defining equation contains one summand for each transition for which place s_i is an input; moreover, as multiparty synchronization is implemented as an atomic sequence of binary synchronizations, for each transition there is the need to elect a leader among its places in the preset that coordinates the actual multiparty synchronization. Some examples are presented to illustrate the approach.

The paper is organized as follows. Section 2 contains some basic background. Section 3 introduces the process calculus Multi-CCS, together with some examples (dining philosophers and concurrent readers/writers). Section 4 defines the operational net semantics for Multi-CCS. Section 5 provides the soundness theorem (p and $Net(p)$ are bisimilar) and the finiteness theorem (for any finite-net process p , $Net(p)$ is finite). Section 6 proves the language expressibility theorem (for any finite reduced P/T net N there exists a finite-net process p_N such that N is isomorphic to $Net(p_N)$). Finally, some conclusions are drawn in Section 7.

2 Background

2.1 Labeled transition systems and bisimulation

Definition 1 A labeled transition system is a triple $TS = (St, A, \rightarrow)$ where St is the set of states, A is the set of labels, $\rightarrow \subseteq St \times A \times St$ is the transition relation. In the following $s \xrightarrow{a} s'$ denotes $(s, a, s') \in \rightarrow$. A rooted transition system is a pair (TS, s_0) where $TS = (St, A, \rightarrow)$ is a transition system and $s_0 \in St$ is the initial state.

Definition 2 A bisimulation between TS_1 and TS_2 is a relation $R \subseteq (St_1 \times St_2)$ such that if $(s_1, s_2) \in R$ then for all $a \in (A_1 \cup A_2)$

- $\forall s'_1$ such that $s_1 \xrightarrow{a} s'_1$, $\exists s'_2$ such that $s_2 \xrightarrow{a} s'_2$ and $(s'_1, s'_2) \in R$
- $\forall s'_2$ such that $s_2 \xrightarrow{a} s'_2$, $\exists s'_1$ such that $s_1 \xrightarrow{a} s'_1$ and $(s'_1, s'_2) \in R$.

If $TS_1 = TS_2$ we say that R is a bisimulation on TS_1 . Two states s and s' are bisimilar, $s \sim s'$, if there exists a bisimulation R such that $(s, s') \in R$.

2.2 Place/Transition Petri nets

Definition 3 Let \mathbb{N} be the set of natural numbers. Given a set S , a finite multiset over S is a function $m : S \rightarrow \mathbb{N}$ such that the set $\text{dom}(m) = \{s \in S \mid m(s) \neq 0\}$ is finite. The multiplicity of s in m is given by the number $m(s)$. The set of all finite multisets over S , $\mathcal{M}_{\text{fin}}(S)$, is ranged over by m . $\mathcal{P}_{\text{fin}}(S)$ is the set of all finite sets over S . We write $m \subseteq m'$ if $m(s) \leq m'(s)$ for all $s \in S$. The operator \oplus denotes multiset union: $(m \oplus m')(s) = m(s) + m'(s)$. The operator \setminus denotes (limited) multiset difference: $(m \setminus m')(s) = \text{if } m(s) > m'(s) \text{ then } m(s) - m'(s) \text{ else } 0$. The scalar product of a number j with m is $(j \cdot m)(s) = j \cdot (m(s))$. A finite multiset m over $S = \{s_1, s_2, \dots\}$ can be also represented as $k_1 s_{i_1} \oplus k_2 s_{i_2} \oplus \dots \oplus k_n s_{i_n}$, where $\text{dom}(m) = \{s_{i_1}, \dots, s_{i_n}\}$ and $k_j = m(s_{i_j})$ for $j = 1, \dots, n$.

Definition 4 A labeled P/T Petri net is a tuple $N = (S, A, T)$, where S is the set of places, A is a set of labels and $T \subseteq \mathcal{M}_{\text{fin}}(S) \times A \times \mathcal{M}_{\text{fin}}(S)$ is the set of transitions. A P/T net is finite if both S and T are finite. A finite multiset over S is called a marking. Given a marking m and a place s , we say that the place s contains $m(s)$ tokens. Given a transition $t = (m, a, m')$, we use the notation $\bullet t$ to denote its preset m , t^\bullet for its postset m' and $l(t)$ for its label a . Hence, transition t can be also represented as $\bullet t \xrightarrow{l(t)} t^\bullet$.

Definition 5 Given a labeled P/T net $N = (S, A, T)$, we say that a transition t is enabled at marking m , written as $m[t]$, if $\bullet t \subseteq m$. The execution of t enabled at m produces the marking $m' = (m \setminus \bullet t) \oplus t^\bullet$. This is written as $m[t]m'$.

A P/T system is a tuple $N(m_0) = (S, A, T, m_0)$, where (S, A, T) is a P/T net and m_0 is a finite multiset over S , called the initial marking. The set of markings reachable from m , denoted $[m]$, is defined as the least set such that $m \in [m]$ and if $m_1 \in [m]$ and, for some transition $t \in T$, $m_1[t]m_2$, then $m_2 \in [m]$. We say that m is reachable if m is reachable from the initial marking m_0 . A P/T system is said to be safe if any place contains at most one token in any reachable marking, i.e. $m(s) \leq 1$ for all $s \in S$ and for all $m \in [m_0]$.

Definition 6 A P/T system $N(m_0) = (S, A, T, m_0)$ is reduced if $\forall s \in S \exists m \in [m_0]$ such that $m(s) \geq 1$, and $\forall t \in T \bullet t \neq \emptyset \wedge \exists m \in [m_0]$ such that $m[t]$.

Definition 7 The interleaving marking graph of $N(m_0)$ is the lts $\text{IMG}(N(m_0)) = ([m_0], A, \rightarrow, m_0)$, where m_0 is the initial state and the transition relation is defined by $m \xrightarrow{l(t)} m'$ iff there exists a transition $t \in T$ such that $m[t]m'$. The P/T systems $N_1(m_1)$ and $N_2(m_2)$ are interleaving bisimilar ($N_1 \sim N_2$) iff there exists a strong bisimulation relating the initial states of $\text{IMG}(N_1(m_1))$ and $\text{IMG}(N_2(m_2))$.

Definition 8 Given two P/T net systems $N_1(m_{0_1})$ and $N_2(m_{0_2})$, we say that N_1 and N_2 are isomorphic if there exists a bijection $f : S_1 \rightarrow S_2$, homomorphically extended to markings, such that $f(m_{0_1}) = m_{0_2}$ and $(m, a, m') \in T_1$ iff $(f(m), a, f(m')) \in T_2$.

3 Multiparty synchronization in CCS

In this section we present Multi-CCS, obtained as a variation over $A^2\text{CCS}$ [12, 11]; the main differences are that in Multi-CCS the parallel operator is associative, and the synchronization relation on sequences is less verbose. Then, two case studies are presented.

3.1 Multi-CCS

Let \mathcal{L} be a denumerable set of channel names, ranged over by a, b, \dots . Let $\overline{\mathcal{L}}$ the set of co-names, ranged over by \bar{a}, \bar{b}, \dots . The set $\mathcal{L} \cup \overline{\mathcal{L}}$, ranged over by α, β, \dots , is the set of visible actions. With $\bar{\alpha}$ we

(Pref)	$\mu.p \xrightarrow{\mu} p$	(S-pref)	$\frac{p \xrightarrow{\sigma} p'}{\underline{\mu}.p \xrightarrow{\mu\sigma} p'}$	
(Sum)	$\frac{p \xrightarrow{\sigma} p'}{p + q \xrightarrow{\sigma} p'}$	(Com)	$\frac{p \xrightarrow{\sigma_1} p' \quad q \xrightarrow{\sigma_2} q'}{p q \xrightarrow{\sigma} p' q'} \quad \text{Sync}(\sigma_1, \sigma_2, \sigma)$	
(Par)	$\frac{p \xrightarrow{\sigma} p'}{p q \xrightarrow{\sigma} p' q}$	(Res)	$\frac{p \xrightarrow{\sigma} p'}{(\nu a)p \xrightarrow{\sigma} (\nu a)p'} \quad a, \bar{a} \notin n(\sigma)$	
(Cong)	$\frac{p \equiv p' \xrightarrow{\sigma} q' \equiv q}{p \xrightarrow{\sigma} q}$	(Cons)	$\frac{p \xrightarrow{\sigma} p'}{C \xrightarrow{\sigma} p'} \quad C \stackrel{def}{=} p$	

Table 1: Operational semantics (symmetric rules for (Sum) and (Par) omitted)

mean the complement of α , assuming that $\bar{\alpha} = \alpha$. Let $Act = \mathcal{L} \cup \bar{\mathcal{L}} \cup \{\tau\}$, such that $\tau \notin \mathcal{L} \cup \bar{\mathcal{L}}$, be the set of actions, ranged over by μ . Action τ denotes an invisible, internal activity. Let \mathcal{C} be a denumerable set of process constants, disjoint from Act , ranged over by A, B, C, \dots . The process terms are generated from actions and constants by:

$$p ::= \mathbf{0} \mid \mu.q \mid \underline{\mu}.q \mid p + p \text{ sequential processes}$$

$$q ::= p \mid q | q \mid (\nu a)q \mid C \text{ processes}$$

where $\mathbf{0}$ is the terminated process, $\mu.q$ is a normally prefixed process where action μ (that can be either an input a , an output \bar{a} or a silent move τ) is first performed and then q is ready, $\underline{\mu}.q$ is a strongly prefixed process where μ is the first action of a transaction that continues with q (provided that q can complete the transaction), $p + p'$ is the sequential process obtained by the alternative composition of sequential processes p and p' , $q | q'$ is the parallel composition of q and q' , $(\nu a)q$ is process q where the (input) name a is made private (restriction), C is a process constant, equipped with a defining equation $C \stackrel{def}{=} q$.

The set \mathcal{P} of *processes* contains those terms which are, w.r.t. process constants they use, *closed* (all the constants possess a defining equation) and *guarded* (for any defining equation $C \stackrel{def}{=} q$, any occurrence of C in q is within a *normally prefixed* subprocess $\mu.q'$ of q). With abuse of notation, \mathcal{P} will be ranged over by p, q, \dots . \mathcal{P}_{seq} is the set of *sequential processes*.

The operational semantics for Multi-CCS is given by the labelled transition system $(\mathcal{P}, \mathcal{A}, \longrightarrow)$, where the states are the processes in \mathcal{P} , $\mathcal{A} = Act^*$ is the set of labels (ranged over by σ), and $\longrightarrow \subseteq \mathcal{P} \times \mathcal{A} \times \mathcal{P}$ is the minimal transition relation generated by the rules listed in Table 1.

We briefly comment on the rules that are less standard. Rule (S-pref) allows for the creation of transitions labeled by non-empty sequences of actions. In order for $\underline{\mu}.q$ to make a move, it is necessary that q can perform a transition, i.e., the rest of the transaction. Hence, $\underline{\mu}.\mathbf{0}$ cannot perform any action. If a transition is labeled by $\sigma = \mu_1 \dots \mu_n$, then all the actions $\mu_1 \dots \mu_{n-1}$ are due to strong prefixes, while μ_n to a normal prefix. Rule (Com) has a side-condition on the possible synchronizability of sequences σ_1 and σ_2 . $\text{Sync}(\sigma_1, \sigma_2, \sigma)$ holds if σ is obtained from an interleaving (possibly with synchronizations) of σ_1 and σ_2 , where the last action of one of the two sequences is to be synchronized, hence reflecting that the subtransaction that ends first signals this fact (i.e., *commits*) to the other subtransaction. Relation Sync is defined by the inductive rules of Table 2. Rule (Res) requires that no action in σ can be a or \bar{a} .

$Sync(\alpha, \bar{\alpha}, \tau)$	$\sigma \neq \varepsilon$	$\sigma \neq \varepsilon$
	$Sync(\alpha\sigma, \bar{\alpha}, \sigma)$	$Sync(\alpha, \bar{\alpha}\sigma, \sigma)$
$Sync(\sigma_1, \sigma_2, \sigma)$	$Sync(\sigma_1, \sigma_2, \sigma)$	$Sync(\sigma_1, \sigma_2, \sigma)$
$Sync(\alpha\sigma_1, \bar{\alpha}\sigma_2, \sigma)$	$Sync(\alpha\sigma_1, \sigma_2, \alpha\sigma)$	$Sync(\sigma_1, \alpha\sigma_2, \alpha\sigma)$
$Sync(\sigma_1, \sigma_2, \sigma)$	$Sync(\sigma_1, \sigma_2, \sigma)$	
$Sync(\tau\sigma_1, \sigma_2, \sigma)$	$Sync(\sigma_1, \tau\sigma_2, \sigma)$	

Table 2: Synchronization relation

$n(\sigma)$ denotes the set of all actions occurring in σ . Rule (Cong) makes use of a structural congruence \equiv on process terms induced by the following three equations:

$$\begin{aligned}
(p|q)|r &= p|(q|r) \\
(va)(p|q) &= p|(va)q \quad \text{if } a \text{ is not free in } p. \\
(va)p &= (vb)(p\{b/a\}) \quad \text{if } b \text{ is not free in } p.
\end{aligned}$$

The first equation is for associativity of the parallel operator; the second one allows for enlargement of the scope of restriction; the last equation is the so-called law of *alpha-conversion*, which makes use of syntactic substitution.¹ Rule (Cong) enlarges the set of transitions derivable from p , as the following example shows. Also, it is necessary to ensure validity of Proposition 14.

Example 1 (Multi-party synchronization) Assume three processes want to synchronize. This can be expressed in Multi-CCS. E.g., consider processes $p = \underline{a}.a.p'$, $q = \bar{a}.q'$ and $r = \bar{a}.r'$ and the whole system $P = (va)((p|q)|r)$. It is easy to see that $P \xrightarrow{\tau} (va)((p'|q')|r')$ (and this can be proved in two ways), so the three processes have synchronized in one single atomic transition. It is interesting to observe that $P' = (va)(p|(q|r))$ could not perform the multiway synchronization if rule (Cong) were not allowed.

Example 2 (Guardedness) We assume that each process constant in a defining equation occurs inside a normally prefixed subprocess $\mu.q$. This will prevent infinitely branching sequential processes. E.g., consider the non legal process $A \stackrel{def}{=} \underline{a}.A + b.0$. According to the operational rules, A has infinitely many transitions leading to 0 , each of the form $a^n b$, for $n = 0, 1, \dots$

Two terms p and q are *interleaving bisimilar*, written $p \sim q$, if there exists a bisimulation R such that $(p, q) \in R$. Observe that $(va)(vb)p \sim (vb)(va)p$, which allows for a simplification in the notation that we usually adopt, namely restriction on a set of names, e.g., $(va, b)p$.

3.2 Case studies

Example 3 (Dining Philosophers) This famous problem, defined by Dijkstra in [6], can be solved in Multi-CCS. Five philosophers seat at a round table, with a private plate and where each of the five forks is shared by two neighbors. Philosophers can think and eat; in order to eat, a philosopher has to acquire both forks that he shares with his neighbors, starting from the fork at his left and then the one at his right.

¹In this paper we use a slightly different definition of syntactic substitution in that $((va)q)\{b/a\} = (vb)q\{b/a\}$ if b is not free in q , so that also the bound name a is converted. This is necessary in the net semantics, in order to be sure that a substitution $\{b/a\}$ will be eventually applied to any inner constant C (defined as p) in q ; the result of $C\{b/a\}$ is a new constant $C\{b/a\} \stackrel{def}{=} p\{b/a\}$. See Example 6 for an application of this idea.

All philosophers behave the same, so the problem is intrinsically symmetric. Clearly a naïve solution would cause deadlock exactly when all five philosophers take the fork at their left at the same time and are waiting for the fork at their right. A simple solution is to force atomicity on the acquisition of the two forks. In order to have a small net model, we consider the case of two philosophers only. The forks can be defined by the constants fork_i :

$$\text{fork}_i \stackrel{\text{def}}{=} \overline{up_i}. \overline{dn_i}. \text{fork}_i \quad \text{for } i = 0, 1$$

The two philosophers can be described as

$$\text{phil}_i \stackrel{\text{def}}{=} \text{think}. \text{phil}_i + \overline{up_i}. up_{i+1}. \text{eat}. \overline{dn_i}. dn_{i+1}. \text{phil}_i \quad \text{for } i = 0, 1$$

where $i + 1$ is computed modulo 2 and the atomic sequence $up_i up_{i+1}$ ensures the atomic acquisition of the two forks. The whole system is

$$DF \stackrel{\text{def}}{=} (\nu L)((\text{phil}_0 | \text{phil}_1) | \text{fork}_0 | \text{fork}_1)$$

where $L = \{up_0, up_1, dn_0, dn_1\}$. Note that the operational semantics generates a finite-state lts for DF .

Example 4 (Concurrent readers and writers) There are several variants of this problem, defined in [4], which can be solved in Multi-CCS. Processes are of two types: reader processes and writer processes. All processes share a common file; so, each writer process must exclude all the other writers and all the readers while writing on the file, while multiple reader processes can access the shared file simultaneously. Assume to have n readers, m writers and that at most $k \leq n$ readers can read simultaneously. A writer must prevent all the k possible concurrent reading operations. A simple solution is to force atomicity on the acquisition of the k locks so that either all are taken or none. To make the presentation simple, assume that $n = 4, k = 3, m = 2$. Each reader process R , each lock process L , each writer W can be represented as follows, where action l stands for lock and u for unlock :

$$R \stackrel{\text{def}}{=} l.\text{read}.u.R \quad L \stackrel{\text{def}}{=} \bar{l}.\bar{u}.L \quad W \stackrel{\text{def}}{=} l.l.l.\text{write}.\bar{u}.\bar{u}.\bar{u}.W$$

$$\text{Sys} \stackrel{\text{def}}{=} (\nu l, u)(((((R|R)|(R|R))|(W|W))|L)|L)|L)$$

It is easy to see that the labeled transition system for Sys is finite-state.

4 Operational Net Semantics

In this section we first describe a technique for building a P/T net for the whole Multi-CCS, starting from a description of its places and of its net transitions. The resulting net $N_{MCCS} = (S_{MCCS}, \mathcal{A}, T_{MCCS})$ is such that, for any $p \in \mathcal{P}$, the net system $N_{MCCS}(\text{dec}(p))$ reachable from the initial marking $\text{dec}(p)$ is a reduced P/T net.

4.1 Places and markings

The Multi-CCS processes are built upon the denumerable set $\mathcal{L} \cup \overline{\mathcal{L}}$, ranged over by α , of visible actions. We assume to have another denumerable set $\mathcal{N} \cup \overline{\mathcal{N}}$ ranged over by δ , of auxiliary *restricted* actions. The set of all actions $\text{Act}' = \mathcal{L} \cup \overline{\mathcal{L}} \cup \mathcal{N} \cup \overline{\mathcal{N}} \cup \{\tau\}$, ranged over by μ with abuse of notation, is used to build the enlarged set of processes we denote with $\mathcal{P}^{\mathcal{N}}$.

The infinite set of places, ranged over by s (possibly indexed), is $S_{MCCS} = \mathcal{P}_{seq}^{\mathcal{N}}$, i.e., the set of all sequential processes over Act' .

Function $\text{dec} : \mathcal{P}^{\mathcal{N}} \rightarrow \mathcal{M}_{fin}(S_{MCCS})$ (see Table 3) defines the decomposition of processes into markings. Agent **0** generates no places. The decomposition of a sequential process p produces one place with

$dec(\mathbf{0}) = \emptyset$	$dec(\mu.p) = \{\mu.p\}$	$dec(\underline{\mu}.q) = \{\underline{\mu}.q\}$
$dec(p + p') = \{p + p'\}$	$dec((\nu a)q) = dec(q\{a'/a\})$	$a' \in \mathcal{N}$ is a new restricted action
$dec(q q') = dec(q) \oplus dec(q')$	$dec(C) = dec(p)$	if $C \stackrel{def}{=} p$

Table 3: Decomposition function

name p . This is the case of $\mu.p$, $\underline{\mu}.p$ and $p + p'$. Parallel composition is interpreted as multiset union; the decomposition of, e.g., $a.\mathbf{0} | a.\mathbf{0}$ produces the marking $a.\mathbf{0} \oplus a.\mathbf{0} = 2a.\mathbf{0}$. The decomposition of a restricted process $(\nu a)q$ generates the multiset obtained from the decomposition of q where the new restricted name $a' \in \mathcal{N}$ is substituted for the bound name a . Finally, a process constant is first unwound once (according to its defining equation) and then decomposed.

It is possible to prove that the decomposition function dec is well-defined by induction on a suitably defined notion of complexity of terms (following [21] page 52). Guardedness (even w.r.t. any kind of prefix) of constants is essential to prove the following obvious fact.

Proposition 9 *For any process $p \in \mathcal{P}^{\mathcal{N}}$, $dec(p)$ is a finite multiset of places.* \square

Note that dec is not injective; e.g., $dec(a.\mathbf{0} | b.\mathbf{0}) = dec(b.\mathbf{0} | a.\mathbf{0})$.

Note that a fresh restricted name a' is to be generated for each of the dec applications on the right-hand-side of the transition schemata we will describe in the next section. So in a recursive term, e.g., $A = (\nu a)(a.A | b.A)$, there may be the need for an unbounded number of fresh names.

4.2 Net transitions

Let $\rightarrow \subseteq \mathcal{M}_{fin}(S_{MCCS}) \times \mathcal{B} \times \mathcal{M}_{fin}(S_{MCCS})$, where $\mathcal{B} = Act'^*$, be the least set of transitions generated by the rules in Table 4.

Let H, K , possibly indexed, range over $\mathcal{M}_{fin}(S_{MCCS})$. In a transition $H \xrightarrow{\sigma} K$, H is the multiset of tokens to be consumed, σ is the label of the transition and K is the multiset of tokens to be produced.

Let us comment the rules. Axiom (pref) states that if one token is present in $\{\mu.q\}$ then a μ -labeled transition is derivable, producing the tokens specified by $dec(q)$. This holds for any μ , i.e., for the invisible action τ , for any visible action α as well as for any restricted action δ . Transition labeled by restricted actions should not be taken in the resulting net, as we restrict ourselves to transitions labeled by sequence on visible actions only (and τ). However, they are useful in producing normal synchronization, as two complementary restricted actions can produce a τ -labeled transition. Rule (s-pref) requires that the premise transition $H \xrightarrow{\sigma} H'$ is derivable by the rules, where H is a submultiset of $dec(q)$. Rule (sum) is as expected. Finally, rule (com) explains how synchronization takes place: it is needed that H and K perform synchronizable sequences σ_1 and σ_2 , producing σ ; here we assume that *Sync* has been extended also to restricted actions in the obvious way.

Note that transitions can be labeled also by restriction actions, while we are interested only in transitions that are labeled on $\mathcal{A} = Act^*$. Hence, the P/T net for Multi-CCS is the triple $N_{MCCS} = (S_{MCCS}, \mathcal{A}, T_{MCCS})$, where the infinite set $T_{MCCS} = \{(H, \sigma, K) \mid H \xrightarrow{\sigma} K \wedge \sigma \in \mathcal{A}\}$ is obtained by filtering out those transitions where no restriction name δ occurs in σ .

(pref)	$\{\mu.q\} \xrightarrow{\mu} dec(q)$	(sum)	$\frac{\{p\} \xrightarrow{\sigma} H}{\{p + p'\} \xrightarrow{\sigma} H}$
(s-pref)	$\frac{H \xrightarrow{\sigma} H'}{\{\mu.q\} \xrightarrow{\mu\sigma} H' \oplus K}$		$H \oplus K = dec(q)$
(com)	$\frac{H \xrightarrow{\sigma_1} H' \quad K \xrightarrow{\sigma_2} K'}{H \oplus K \xrightarrow{\sigma} H' \oplus K'}$		$Sync(\sigma_1, \sigma_2, \sigma)$

Table 4: Rules for net transitions (symmetric rule for (sum) omitted).

Proposition 10 *Let $t = H \xrightarrow{\sigma} H'$ be a transition. Let p be such that $dec(p) = H \oplus K$ and let t be enabled at $dec(p)$. Then $H' \oplus K = dec(p')$ for some p' .*

Proof: By induction on the definition of $dec(p)$ and then on the proof of t . □

Given a process p , the P/T system associated to p is the subnet of N_{MCCS} reachable from the initial marking $dec(p)$. We indicate with $Net(p)$ such a subnet.

Definition 11 *Let p be a process. The P/T system associated to p is $Net(p) = (S_p, A_p, T_p, m_0)$, where $m_0 = dec(p)$ and*

$$\begin{aligned}
 S_p &= \{s \in S_{MCCS} \mid \exists m \in [m_0] (m(s) > 0)\} \\
 T_p &= \{t \in T_{MCCS} \mid \exists m \in [m_0] \text{ s.t. } m[t]\} \\
 A_p &= \{\sigma \in \mathcal{A} \mid \exists t \in T_p, \sigma = l(t)\}
 \end{aligned}$$

The definition above suggests a way of generating $Net(p)$ with an algorithm in least-fixpoint style. Start by $dec(p)$ and then apply the rules in Table 4 in order to produce the set of transitions (labeled on \mathcal{A}) executable from $dec(p)$ in one step. This will also produce possible new places to be added to the current set of places. Then repeat until no new places are added and no new transitions are derivable; hence, this algorithm ends only for finite nets.

The following facts are obvious by construction:

Proposition 12 *For any $p \in \mathcal{P}$,*

- $Net(p)$ is a reduced (see Definition 6) P/T net.
- $Net(p) \sim N_{MCCS}(dec(p))$.

4.3 Case Studies

Example 5 (Semi-counter) *A semi-counter process, i.e., a counter that cannot test for zero, can be described by the infinite-state process $A \stackrel{def}{=} up.(down.\mathbf{0} \mid A)$. Observe that $dec(A) = \{up.(down.\mathbf{0} \mid A)\}$. The only enabled transition is $dec(A) \xrightarrow{up} down.\mathbf{0} \oplus up.(down.\mathbf{0} \mid A)$. Then, also transition $down.\mathbf{0} \xrightarrow{down} \mathbf{0}$ is derivable. The finite P/T net $Net(A)$ is reported in Figure 1.*

Example 6 (Counter with test for zero) *As an example of a CCS process that cannot be modeled by a finite P/T net, consider the following specification of a (real) counter, as given in [23].*

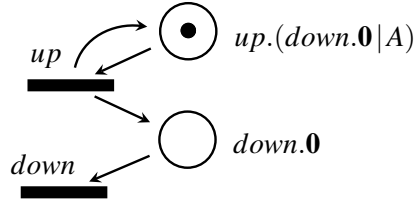
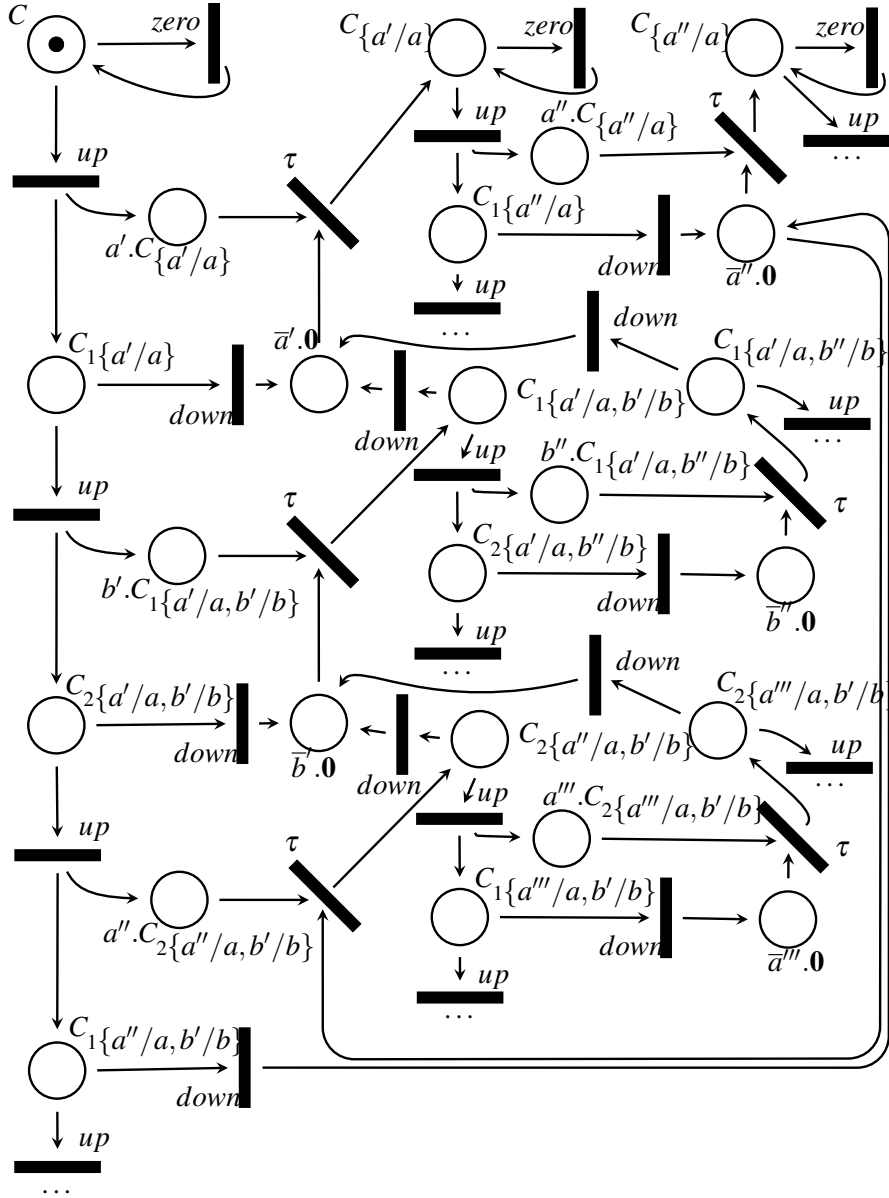


Figure 1: The P/T system for a semi-counter.

Figure 2: The initial fragment of the P/T system for counter C .

$$\begin{aligned}
C &\stackrel{def}{=} \text{zero}.C + \text{up}.((va)(C_1 | a.C)) \\
C_1 &\stackrel{def}{=} \text{down}.\bar{a}.\mathbf{0} + \text{up}.((vb)(C_2 | b.C_1)) \\
C_2 &\stackrel{def}{=} (\text{down}.\bar{b}.\mathbf{0} + \text{up}.((va)(C_1 | a.C_2)))
\end{aligned}$$

An initial fragment of the infinite P/T net $\text{Net}(C)$ is reported² in Figure 2, where successive unfoldings are due to syntactic substitutions applied to constants that generate new places. Note also the peculiar way substitution is applied to restricted terms.

Example 7 (Dining Philosophers) Consider the system DF of Example 3. The marking $\text{dec}(DF)$ is composed of the four places³ $s_1 = \text{phil}_0$, $s_2 = \text{phil}_1$, $s_3 = \text{fork}_0$ and $s_4 = \text{fork}_1$. Initially, the two philosophers can think on their own:

$$s_1 \xrightarrow{\text{think}} s_1 \text{ and } s_2 \xrightarrow{\text{think}} s_2$$

or can compete for the acquisition of the two forks:

$$s_1 \oplus s_3 \oplus s_4 \xrightarrow{\tau} s'_1 \oplus s'_3 \oplus s'_4 \text{ and}$$

$$s_2 \oplus s_3 \oplus s_4 \xrightarrow{\tau} s'_2 \oplus s'_3 \oplus s'_4$$

where $s'_1 = \text{phil}'_0$, $s'_2 = \text{phil}'_1$, $s'_3 = \overline{\text{down}_0}.\text{fork}_0$, $s'_4 = \overline{\text{down}_1}.\text{fork}_1$

with, for $i = 0, 1$, $\text{phil}'_i = \text{eat}.\underline{\text{down}_i}.\text{down}_{i+1(\text{mod}2)}.\text{phil}_i$. Now two further alternative transitions are derivable, namely:

$$s'_1 \xrightarrow{\text{eat}} s''_1 \text{ and } s'_2 \xrightarrow{\text{eat}} s''_2$$

where $s''_1 = \text{phil}''_0$, $s''_2 = \text{phil}''_1$, with, for $i = 0, 1$, $\text{phil}''_i = \underline{\text{down}_i}.\text{down}_{i+1(\text{mod}2)}.\text{phil}_i$. Finally,

$$s''_1 \oplus s'_3 \oplus s'_4 \xrightarrow{\tau} s_1 \oplus s_3 \oplus s_4 \text{ and}$$

$$s''_2 \oplus s'_3 \oplus s'_4 \xrightarrow{\tau} s_2 \oplus s_3 \oplus s_4$$

and we are back to the initial marking $\text{dec}(DF)$. The resulting $\text{Net}(DF)$ is reported in Figure 3(a). Note that the two philosophers can never eat at the same time, i.e., in no reachable marking m we have that $m(s'_1) = 1 = m(s'_2)$.

Example 8 (Concurrent readers and writers) Let us consider Sys of Example 4. The multiset $\text{dec}(\text{Sys})$ is $4\text{rd} \oplus 3\text{lk} \oplus 2\text{wr} \oplus (\text{vl}) \oplus (\text{vu})$, where $\text{rd} = l.\text{read}.u.R$, $\text{lk} = \bar{l}.\bar{u}.L$ and $\text{wr} = \underline{l}.\underline{l}.\text{write}.\underline{u}.\underline{u}.W$. One of the two possible initial transitions is $\text{wr} \oplus 3\text{lk} \xrightarrow{\tau} \text{wr}' \oplus 3\text{lk}'$, where $\text{wr}' = \text{write}.\underline{u}.\underline{u}.W$ and $\text{lk}' = \bar{u}.L$. After such a transition, no reader can read, as all the locks are busy. The other possible initial transition is $\text{rd} \oplus \text{lk} \xrightarrow{\tau} \text{rd}' \oplus \text{lk}'$, where $\text{rd}' = \text{read}.u.R$. The resulting P/T net $\text{Net}(\text{Sys})$ is depicted in Figure 3(b).

5 Properties of the net semantics

In this section, we present some results about the net semantics we have defined. First we give a soundness result, namely that the interleaving marking graph associated to $\text{Net}(p)$ for any Multi-CCS term p is bisimilar to its transition system. Then we discuss finiteness conditions on the net semantics. In particular, we single out a subclass of Multi-CCS processes whose semantics always generates finite P/T nets. This subclass, we call *finite-net* processes, is rather rich, as the parallel operator is allowed to occur inside the body of recursively defined constants. Hence, finite-net processes may be infinite-state processes, (i.e., the associated labeled transition system may contain infinitely many states), as illustrated in Example 5.

²For brevity, we associate to a place the name of a constant instead of its definition, e.g. place C should be called $\text{zero}.C + \text{up}.((va)(C_1 | a.C))$.

³Again, for brevity, we associate to a place the name of a constant instead of its definition, e.g. $s_1 = \text{phil}_0$ while it should be $s_1 = \text{think}.\text{phil}_0 + \underline{\text{up}_0}.\text{up}_1.\text{eat}.\underline{\text{dn}_0}.\text{dn}_1.\text{phil}_0$.

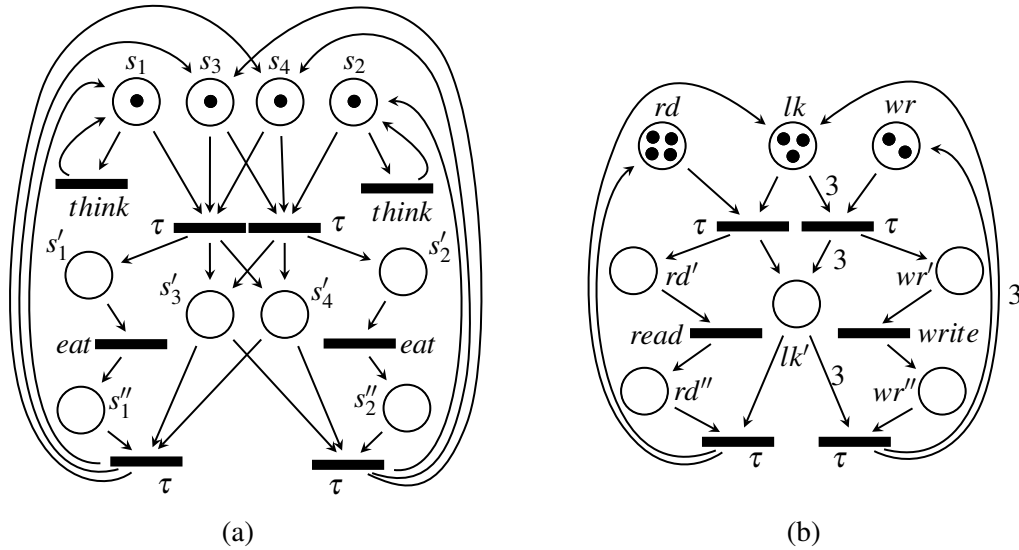


Figure 3: (a) The net for two dining philosophers. (b) The net for concurrent readers/writers.

5.1 Soundness

Proposition 13 For any process $p \in \mathcal{P}$, if $p \xrightarrow{\sigma} p'$ then there exists $t \in T_p$ such that $\text{dec}(p)[t]K$ with $l(t) = \sigma$ and $K \sim \text{dec}(p')$.

Proof: By induction on the proof of $p \xrightarrow{\sigma} p'$. □

Proposition 14 For any process $p \in \mathcal{P}$, if there exists $t \in T_p$ such that $\text{dec}(p)[t]K$ with $l(t) = \sigma$, then there exists p' such that $p \xrightarrow{\sigma} p'$ and $K = \text{dec}(p')$.

Proof: By induction on (the definition of) $\text{dec}(p)$ and then by induction on the proof of t . □

Theorem 15 For any process $p \in \mathcal{P}$, $p \sim \text{dec}(p)$.

Proof: Relation $R = \{(p, \text{dec}(q)) \mid p, q \in \mathcal{P}, \text{dec}(p) \sim \text{dec}(q)\}$ is a bisimulation, due to Proposition 13 (together with Proposition 10) and Proposition 14. □

5.2 Finiteness

The net semantics often generates finite nets. However, the generation of an infinite system may be due to one of the following three facts. First, the decomposition rule for restriction requires the generation of a fresh name; hence, if this operator lies inside a recursive definition, an infinite set of fresh names (i.e., of places) may be required. Second, we have to impose a finite bound to the number of constants that can be used in a process definition. E.g., process $b.A_0$, with the family of process constants $A_i \stackrel{\text{def}}{=} a_i.A_{i+1}$ for $i \in \mathbb{N}$, is not allowed. Third, as the synchronization relation is too generous (it may produce infinitely many transitions even for a net with finitely many places, as the following example shows), we have to impose a restriction over *Sync*, that disables transactional communication but allows for multi-party synchronization.

Example 9 Consider $B \stackrel{\text{def}}{=} \underline{a}.\bar{a}.(B|B)$. $\text{Net}(B)$ has just one place $p = \underline{a}.\bar{a}.(B|B)$, but infinitely many transitions! The only possible initial net transition is $p \xrightarrow{a\bar{a}} 2p$. Now transition $2p \xrightarrow{a\bar{a}} 4p$ is possible, and then $4p \xrightarrow{a\bar{a}} 8p$, and so on ad infinitum.

Definition 16 The finite-net Multi-CCS processes are the processes generated by the following syntax

$$\begin{aligned} s &::= \mathbf{0} \mid \mu.t \mid \underline{\mu}.t \mid s + s \\ t &::= s \mid t \mid t \mid C \\ p &::= t \mid (\nu a)p \mid p \mid p \end{aligned}$$

where a constant C has associated a term of type t , i.e., $C \stackrel{def}{=} t$ and the number of constants involved in any process definition is always finite.

The semantics of finite-net Multi-CCS is the same as provided for Multi-CCS in Tables 1 and 2, with the following additional constraint on rule (Com): $\text{Sync}(\sigma_1, \sigma_2, \sigma)$ is applicable only if $|\sigma_1| = 1$ or $|\sigma_2| = 1$. \square

Theorem 17 Let p be a finite-net process. Then the subnet $\text{Net}(p)$ associated to p is finite. \square

6 A process term for any finite P/T net

Now the converse problem: given a finite P/T system $N(m_0)$, can we single out a finite-net process $p_{N(m_0)}$ such that $Cl(p_{N(m_0)})$ and $N(m_0)$ are isomorphic? The answer is positive, hence providing a language for finite P/T Petri nets.

The translation from nets to processes we present takes a restricted name y_i for any place s_i ; this is used to distinguish syntactically all the places, so that no fusion is possible when applying the reduced net reverse translation. Moreover, it considers a restricted name x_j for each transition t_j , that is used to synchronize all the components that participate in t_j . The constant C_i associated to a place s_i has a summand for each transition which s_i is in the preset of. Among the many places in the preset of t_j , the one connected with an arc of minimal weight (and if more than one is so, then the one with minimal index) plays the role of *leader* of the multiparty synchronization (i.e., the process performing the atomic sequence of inputs x_j to be synchronized with single outputs \bar{x}_j performed by the other participants).

Definition 18 Let $N(m_0) = (S, A, T, m_0)$, with $S = \{s_1, \dots, s_n\}$ and $T = \{t_1, \dots, t_v\}$. Function $\text{INet}(N(m_0))$ from finite P/T systems to finite-net processes is defined as (for fresh x_i and y_j)

$$\text{INet}(N(m_0)) = (\nu x_1 \dots \nu x_v)(\nu y_1 \dots \nu y_n) \underbrace{(C_1 \mid \dots \mid C_1)}_{m_0(s_1)} \mid \dots \mid \underbrace{(C_n \mid \dots \mid C_n)}_{m_0(s_n)}$$

where each C_i has a defining equation

$$C_i \stackrel{def}{=} c_i^1 + \dots + c_i^{p_i} + y_i.\mathbf{0}$$

where p_i is the size of $s_i^\bullet = \{t_{i_1}, \dots, t_{i_{p_i}}\} \subseteq T$ such that $s_i \in \text{dom}(\bullet t)$ for each $t \in s_i^\bullet$. Let $d_{ij} = \sum_k (\bullet t_{ij}(s_k)) - 1$ and $a_{ij} = l(t_{ij})$. Then, each c_i^j is equal to

- $a_{ij}.\Pi_{ij}$ if $d_{ij} = 0$ (no synchronization as $\bullet t_{ij} = s_i$);
- $\bar{x}_{ij}.\mathbf{0}$ if the previous condition does not hold, and $\bullet t_{ij}(s_i) > \bullet t_{ij}(s_{i'})$ for some i' or $\bullet t_{ij}(s_i) = \bullet t_{ij}(s_{i'})$ for some $i' < i$ (i.e., s_i is not the leader for the synchronization on t_{ij});
- $\underbrace{\bar{x}_{ij} \dots \bar{x}_{ij}}_{d_{ij}}.a_{ij}.\Pi_{ij}$ if the previous conditions do not hold (i.e., s_i is the leader), and $\bullet t_{ij}(s_i) = 1$; if $a_{ij} = \tau$, c_i^j is simplified to $\underbrace{\bar{x}_{ij} \dots \bar{x}_{ij}}_{d_{ij}-1}.x_{ij}.\Pi_{ij}$;
- $\bar{x}_{ij}.\mathbf{0} + \underbrace{\bar{x}_{ij} \dots \bar{x}_{ij}}_{d_{ij}}.a_{ij}.\Pi_{ij}$ otherwise (i.e., s_i is the leader and the arc has weight > 1).

Finally, each Π_{ij} is defined as $\Pi_{ij} = \underbrace{C_1 \mid \dots \mid C_1}_{t_{ij}^\bullet(s_1)} \mid \dots \mid \underbrace{C_n \mid \dots \mid C_n}_{t_{ij}^\bullet(s_n)}$.

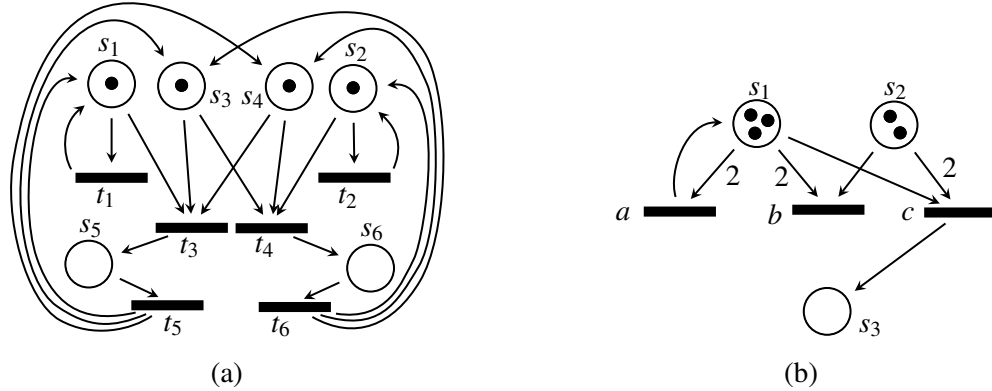


Figure 4: (a) Alternative two philosophers' net. (b) A simple net

Remark: (CCS nets) Let us call *CCS nets* the class of P/T nets where transitions have only one input arc (with weight 1) or two input arcs (with weight 1) but labelled by τ . It is not difficult to see that, given a CCS net $N(m_0)$ the resulting process term $INet(N(m_0))$ is a finite-net CCS terms (i.e., a term without strong prefixing).

Example 10 Consider the net N depicted in Figure 4(a), where we assume that $l(t_1) = l(t_2) = \text{think}$, $l(t_3) = l(t_4) = \tau$ and $l(t_5) = l(t_6) = \text{eat}$. Clearly, it is a different solution to the dining philosophers problem, where forks (places s_3 and s_4) are resources that are consumed and then regenerated. Applying the translation above, we obtain the finite-net process $INet(N(m_0)) = (\nu x_1 \dots x_6)(\nu y_1 \dots y_6)(C_1 | C_2 | C_3 | C_4)$ where

$$\begin{aligned} C_1 &\stackrel{\text{def}}{=} \text{think}.C_1 + \underline{x_3}.x_3.C_5 + y_1.\mathbf{0} & C_2 &\stackrel{\text{def}}{=} \text{think}.C_2 + \underline{x_4}.x_4.C_6 + y_2.\mathbf{0} \\ C_3 &\stackrel{\text{def}}{=} \overline{x_3}.\mathbf{0} + \overline{x_4}.\mathbf{0} + y_3.\mathbf{0} & C_4 &\stackrel{\text{def}}{=} \overline{x_3}.\mathbf{0} + \overline{x_4}.\mathbf{0} + y_4.\mathbf{0} \\ C_5 &\stackrel{\text{def}}{=} \text{eat}.(C_1 | C_3 | C_4) + y_5.\mathbf{0} & C_6 &\stackrel{\text{def}}{=} \text{eat}.(C_2 | C_3 | C_4) + y_6.\mathbf{0} \end{aligned}$$

Note that C_3 and C_4 differ for the last summand only. If the restricted names y_3 and y_4 were omitted, $Net(INet(N(m_0)))$ would be a different net where places s_3 and s_4 are fused in a new place with two tokens.

$INet(N(m_0))$ generates an infinite-state labeled transition system, because of the nesting of parallel operator inside recursively defined constants. However, its behavior is actually finite: indeed, it generates a finite safe P/T net, hence with a finite interleaving marking graph, which is bisimilar to its infinite-state labeled transition system.

Example 11 Consider the net $N(m_0)$ of Figure 4(b). Applying the translation above, we obtain the finite-net process $INet(N(m_0)) = (\nu x_1 x_2 x_3)(\nu y_1 y_2 y_3)(C_1 | C_1 | C_2 | C_2)$ where

$$\begin{aligned} C_1 &\stackrel{\text{def}}{=} \overline{x_1}.\mathbf{0} + \underline{x_1}.a.C_1 + \overline{x_2}.\mathbf{0} + \underline{x_3}.x_3.c.C_3 + y_1.\mathbf{0} \\ C_2 &\stackrel{\text{def}}{=} \underline{x_2}.x_2.b.\mathbf{0} + \overline{x_3}.\mathbf{0} + y_2.\mathbf{0} & C_3 &\stackrel{\text{def}}{=} y_3.\mathbf{0} \end{aligned}$$

Theorem 19 Let $N(m_0)$ be a finite reduced system. Then, $Net(INet(N(m_0)))$ is isomorphic to $N(m_0)$. \square

Corollary 20 Let $N(m_0)$ be a finite reduced CCS net. Then, $INet(N(m_0))$ is a CCS process term and $Net(INet(N(m_0)))$ is isomorphic to $N(m_0)$. \square

7 Conclusion

The class of finite-net Multi-CCS processes represents a language for describing finite P/T nets. This is not the only language expressing P/T nets: the first (and only other) one is Mayr's PRS [15], which however is rather far from a typical process algebra as its basic building blocks are rewrite rules (instead of actions) and, for instance, it does not contain any scope operator like restriction or hiding. We think the language we have identified can be used in order to cross-fertilize the areas of process calculi and Petri nets. In one direction, it opens, e.g., the problem of finding axiomatizations of Petri nets behaviours. For instance, net isomorphism induces a lot of equations over Multi-CCS terms. Just to mention a few, parallel composition is associative, commutative with $\mathbf{0}$ as neutral element, terms that differ only for alpha-conversion of bound names are identified, the sum operator is associative, commutative and, if the sequential term p is not $\mathbf{0}$, then also $p + \mathbf{0} = p$ and $p + p = p$ hold. Even if the problem of finding a complete set of axioms characterizing net isomorphism is probably out-of-reach, nonetheless, the axioms we have identified are interesting as they include those forming the structural congruence for CCS [20], hence validating their use. On the other direction, Petri net theory can offer a lot of support to process algebra. Some useful properties are decidable for finite P/T nets (e.g., reachability, liveness, coverability – see e.g., [22] – model-checking of linear time μ -calculus formulae [7]) and so also the (infinite-state systems of) finite-net Multi-CCS processes can be checked against these properties. Moreover, P/T nets are equipped with non-interleaving semantics, where parallel composition is not reduced to sum and prefixing, and these semantics can be used fruitfully to check causality-based properties, useful, e.g., in error recovery.

As a final remark, we want to stress that our net semantics is the first one based on unsafe labeled P/T nets for a rich process algebra including CCS as a subcalculus. Indeed, our net semantics improves over previous work. Goltz's result [8, 9] are limited to CCS without restriction; we define our net semantics in a different style (operational) and additionally we cope with restriction and strong prefixing. Degano, De Nicola, Montanari [5] and Olderog's approach [21] is somehow complementary in style, as it builds directly over the SOS semantics of CCS. Their construction generates *safe* P/T nets which are finite only for regular CCS processes (i.e., processes where restriction and parallel composition cannot occur inside recursion). Moreover, this approach has never been applied to a process algebra whose labeled operational semantics is defined modulo a structural congruence. Similar concerns are for PBC [2], whose semantics is given in terms of safe P/T nets. Nonetheless, PBC can express "programmable" multiway synchronization by means of its relabeling operators (somehow similar to Multi-CCS), and so, in principle, if equipped with an unsafe semantics it might also serve as a language expressing general P/T nets. On the contrary, we conjecture that it is not possible to obtain a representation theorem such as Theorem 19 based on CSP [14].

Our work is somehow indebted to the earlier work of Busi & Gorrieri [3] on giving labeled net semantics to π -calculus in terms of P/T nets with inhibitor arcs; our solution simplifies this approach for CCS and Multi-CCS because we do not need inhibitors. In particular, already in that paper it is observed that finite-net π -calculus processes originate finite net P/T net systems (with inhibitor arcs). Similar observations on the interplay between parallel composition and restriction in recursive definitions, in different contexts, has been done also by others, e.g., [1]. Also important is the work of Meyer [16, 17] in providing an unlabeled P/T net semantics for a fragment of π -calculus; the main difference is that his semantics may offer a finite net representation also for some processes where restriction occurs inside recursion, but the price to pay is that the resulting net semantics may be not correct from a causality point of view. We conjecture that his technique is not applicable to Multi-CCS.

Future work will be devoted to define compositional (denotational in style) unsafe net semantics for

Multi-CCS, generalizing work of Goltz [8] and Taubner [23].

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